



CMX998 Loop Noise Calculator

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1 INTRODUCTION & ACKNOWLEDGMENT

This report follows the work of D.W.Reeves in his 'Cartesia6a' document which discusses Cartesian Loop theory and also contains a time-expired program which would simulate a Cartesian Feedback Loop. Herein we revisit the theory discussed in 'Cartesia6a' as well as go through the new mathematical model written to simulate the Cartesian Feedback Loop.

The model is created in Octave (<https://www.octave.org/download.html>) and can be accessed by running the file named: CMX998_LNC.m. This is contained in a folder named CMX_998_Loop_Noise_Calculator which contains the instructions required to run the program. A Matlab version is also available.

2 INTRODUCTORY MATERIAL

Referring to Figure 2, essentially what the circuit we will look at does is take an electronic signal at X, change its frequency and power, then send out a radio signal at Y. Along with our signal we find that there is some unwanted noise which can be observed in Figure 1. In order for our signal to be useful we need the level of the signal to be much greater than the level of the noise, so we will be calculating how much noise this circuit generates. We measure the power of the signal and the noise in dB (decibels) for relative measurements or dBm (which is dBW + 30) for absolute power levels.

Within the circuit there are lots of different components which have two properties that are important in our calculations: power gain and noise figure. The power gain is the number of dB that a component adds to any signal or noise that passes through it. The noise figure is the number of dB that the 'signal quality' reduces by when a signal with noise passes through it. To appreciate this a bit more we must define the noise figure of a component:

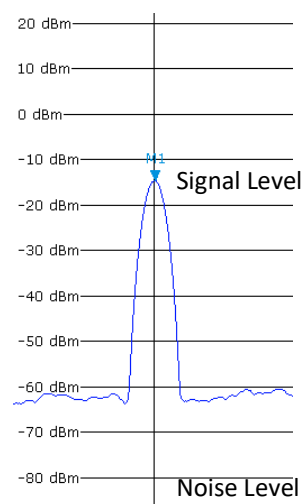


Figure 1: Example of a signal.

$$\text{Noise.Figure} = (\text{Signal.Input} - \text{Noise.Input}) - (\text{Signal.Output} - \text{Noise.Output})$$

From this definition we can see that noise figure tells us the difference between the signal to noise level at the input and output of a component. It should be noted that this number should never be negative. All of the inputs to our model will be in dB as these are nice and simple values to quote, however we perform all our calculations in a different form, which we will call the 'non-dB form'.

To do this we need to convert between our two forms and we do this like so: If we say X is a dB value and Y is the corresponding non-dB value, we can convert between the two using the following formulae:

$$X = 10 \cdot \log_{10}(Y)$$

$$Y = 10^{(X/10)}$$

This will be reiterated later. We will continue to refer to our new power gain values as the 'power gain' and if the dB values are to be considered this will be made clear by a reference to the 'dB power gain'. Less confusingly the noise figure now becomes the noise factor. On the left hand side of the Cartesian Loop diagram (see Figure 2 in the start of the next section) we observe that there are two parallel paths (baseband in-phase and quadrature format signals). We don't need to worry about this

because the signal is mathematically the same and can be treated as if there was only one path like there is on the right hand side.

We should also note that whenever we send a signal into a system or a component there will also be some noise attached to this. There is a minimum noise level called the thermal noise which will appear later in this paper.

In our graphs we will plot frequency on the x-axis. This frequency is called the 'offset'. To explain what this is we must first know the frequency that the signal peak, as we saw in Figure 1, is measured at. We call this the centre frequency. The offset is the frequency measured in reference to the centre frequency, i.e. it's a distance from the centre frequency.

There is a component called a phase shifter in the circuit which introduces a variable angle, ϑ , into the local oscillator path to the down converter. Essentially all this does is allows the phase between the up converter and down converter paths to be varied. Also, we introduce a time delay which basically allows us to pause the signals progress in the feedback loop. We won't go into these any further at present but they will be a factor in our calculations.

3 CARTESIAN FEEDBACK LOOP THEORY

The Cartesian Loop that we shall look at is shown in Figure 2.

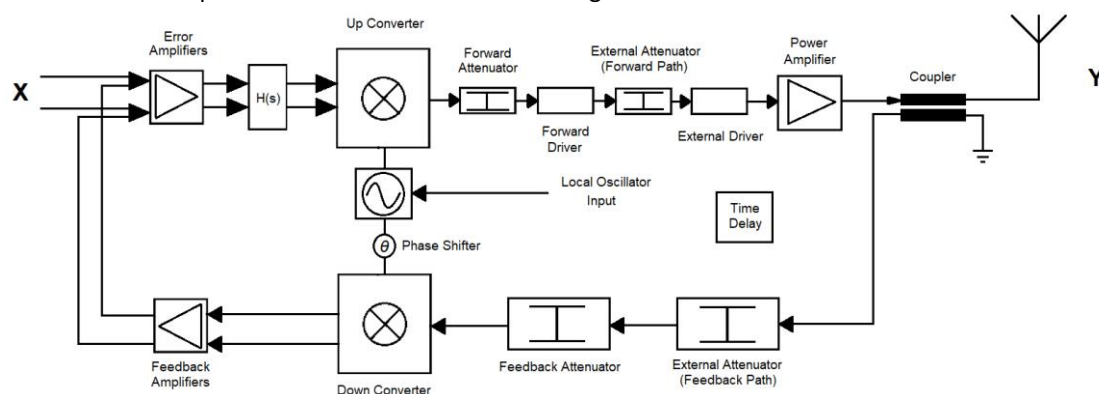


Figure 2: Cartesian Loop Using CMX998 Block Diagram.

We have a baseband input at **X** and our RF output is at **Y**. The arrangement consists of both a forward and a feedback path. In the forward path we have summing Error Amplifiers, a Transfer function $H(s)$, an Up Converter Mixer, two forward path Attenuators and Drivers, and a Power Amplifier. The feedback path contains two Attenuators, a Down Converter Mixer and additional amplifiers. Between the forward and feedback path we have a coupler which feeds some of the forward path signal into the feedback path in order to achieve our Cartesian Loop. We also have a Local Oscillator (LO), Phase Shifter and a Time Delay, the LO will introduce additional noise to the system. We will give the following definitions to the components in our circuit:

ErrAmp:	Gain = ErrAmp1.Gain	F = ErrAmp.Noise
UpConv:	Gain = UC.Gain	F = UC1.Noise
FwdAtten:	Gain = FwdAtten.Gain	F = FwdAtten.Noise
FwdDriver:	Gain = FwdDriver.Gain	F = FwdDriver.Noise
ExtAttenFwd:	Gain = ExtAttenFwd.Gain	F = ExtAttenFwd.Noise
ExtDriver:	Gain = ExtDriver.Gain	F = ExtDriver.Noise
PA:	Gain = PA.Gain	F = PA.Noise
Coupler:	Gain = CouplerFwd.Gain	

	Gain = Coupler.Gain	F = Coupler.Noise
LO:		F = LO.Noise
Phase Shifter:		F = PS.Noise
ExtAttenFb:	Gain = ExtAttenFb.Gain	F = ExtAttenFb.Noise
FbAtten:	Gain = FbAtten1.Gain	F = FbAtten.Noise
DownConv:	Gain = DC1.Gain	F = DC1.Noise
FbAmp:	Gain = FbAmp.Gain	F = FbAmp.Noise

Note: In the forward direction the coupler gives a small loss of power but is assumed to be noiseless. It is assumed that the Local Oscillator and Phase Shifter have no gain.

Attenuators, by definition, have loss and therefore their gain is the inverse of their loss.

All calculations are done in the non-dB form. If we say X is a dB value and Y is the corresponding non-dB value, we can convert between the two using the following formulae:

$$X = 10 \cdot \log_{10}(Y)$$

$$Y = 10^{(X/10)}$$

4 TRANSFER FUNCTION, H(S)

The transfer function, H(s), is a multiplication term which controls the gain of the preceding Amplifier and it's assumed to be noiseless. We can therefore express the linear power gain of the Error Amplifier as:

$$\text{ErrAmp:} \quad \text{ErrAmp.Gain} = \text{ErrAmp1.Gain} \cdot |H(s)|^2$$

The transfer function can take the form of either a simple integrator or a low-pass filter consisting of a capacitor-resistor network (as below) which is associated with an error (or differential) amplifier (often referred to as an operational amplifier or op-amp).

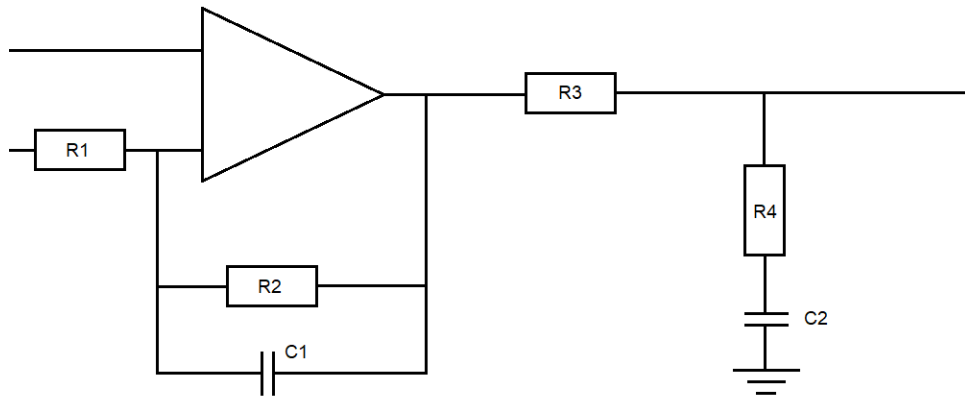


Figure 3: Resistor-Capacitor Network around the Error Amplifier.

We can then derive the transfer function for this network to be:

$$H(s) = (s \cdot t_3 + 1) / ((s \cdot t_1 + \text{FILTER}) \cdot (s \cdot (t_2 + t_3) + 1))$$

where $s = 2 \cdot \pi \cdot f$, $t_1 = C1 \cdot R2$, $t_2 = C2 \cdot R3$, $t_3 = C2 \cdot R4$ and FILTER=0 if an integrator is used, FILTER=1 if a Low-Pass Filter is used.

Note: The t_m values are called time constants.

5 LOCAL OSCILLATOR (LO) NOISE

The variation of LO Noise with frequency can be modelled. It's defined by three variables LO.N1, LO.N2 and F1. LO.N1 and LO.N2 are noise terms measured in dBc/Hz (referenced to the converter input signal level that the LO is attached to) and F1 is the frequency measured in Hz. We also define a second frequency, $F2 = F1 \cdot 10^{((LO.N1 - LO.N2)/20)}$, and we can now model the LO noise thus:

$f < F1$: LO1.Noise = LO.N1 in dBc/Hz
 $f > F2$: LO1.Noise = LO.N2 in dBc/Hz
 $F1 < f < F2$: LO1.Noise = LO.N1 - $20 \cdot (\log_{10}(f) - \log_{10}(F1))$ in dBc/Hz.

This can be plotted on a graph with a logarithmic frequency scale as below:

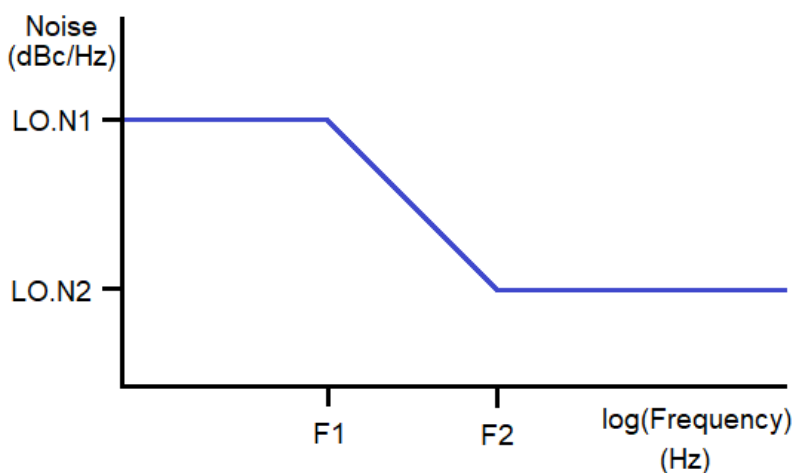


Figure 4: Graph of the Local Oscillator Noise profile.

6 MIXER NOISE

The noise of the Up-Converter mixer is a combination of the Noise generated by the LO, the Phase Shifter¹ and the Up-Converter itself. Its arrangement is modelled below:

¹ In CMX998 the phase-shifter is in the LO path to the down-converter so the noise parameter in the forward path should be set to 0 dB.

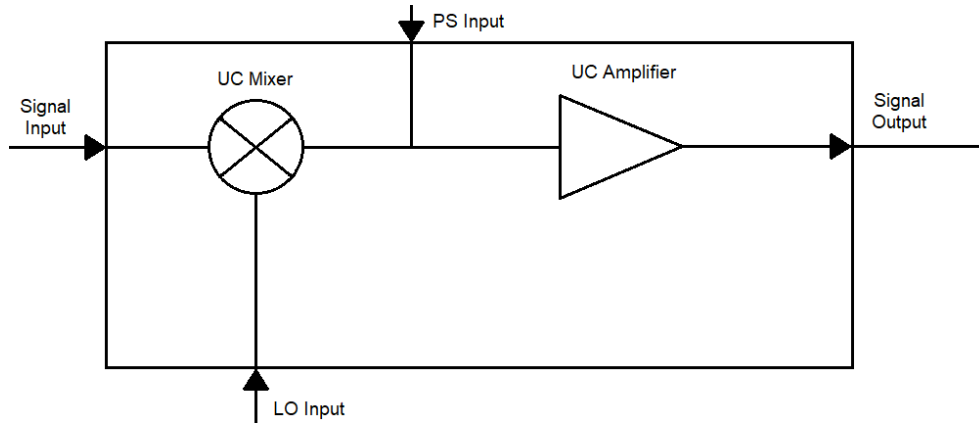


Figure 5: Diagram of the Up Converter in CMX998.

We assume that the LO and the Phase Shifter do not contribute any gain but they² will contribute noise. Hence we can find the Noise Factor of the mixer to be as follows:

$$\text{UC.Noise} = \text{LO.Noise} * \text{UC.Input} + \text{UC1.Noise} + (\text{PhaseShifter.Noise} - 1)$$

where UC.Input = Input power at the Up Converter.

The derivation of this can be found in Appendix A of this document, page 12.

Given that the LO Noise is given in dBc/Hz, its contribution to the mixer noise is relative to the mixer's input signal level. The effect of the Phase Shifter noise is to raise the overall mixer noise floor.

The Down Converter is modelled in the same way as the Up Converter, the only exception being the removal of the Phase Shifter. This means the noise factor of the Down Converter can be expressed as:

$$\text{DC.Noise} = \text{LO.Noise} * \text{DC.Input} + \text{DC1.Noise}$$

7 TIME DELAY AND PHASE

The signal at the Up Converter is translated to an RF frequency which has a phase of the following form:

$$(f_c + f) * t$$

Where f_c is the centre frequency (half the LO input frequency) and f is the offset frequency. t is for time. A phase term, ϑ , is added to the LO signal which introduces a phase shift to the resulting Up Converter output signal. This takes the following form:

$$((f_c + f) * t + \vartheta)$$

Once the RF signal gets to the Down Converter it will have incurred a time delay, τ , and the signal phase will now have the following form:

$$((f_c + f) * (t - \tau) + \vartheta)$$

² In CMX998 the phase shifter does not contribute to the forward path noise. However, mathematically we can include the Phase Shifter in either the forward or feedback path.

The signal is then demodulated by the LO signal ($f_c * t$) and the resulting signal sent back to the error amplifiers will have a phase of the form:

$$(f * t - (f_c + f) * \tau + \vartheta)$$

The signal may then be expressed in exponential form which will allow us to incorporate it in our model later.

$$\exp(i * f * t) * \exp(i * (\vartheta - (f_c + f) * \tau))$$

The first term represents the time domain signal. The second term describes the effect of the loops phase shift and delay properties.

8 FEEDBACK AMPLIFIER GAIN

The gain of the feedback path in our circuit is constructed in an abstract way. The gain of the external attenuator (Feedback Path) is fixed but the feedback attenuator gain can be varied between 0 dB and -25 dB in the CMX998 GUI. This value then sets the gain for both the Feedback Attenuator and the Down Converter as follows. The Down Converter has an intrinsic gain property of $DC1.Gain = Y$ dB. We set the Feedback Attenuator to have gain $FbAtten1.Gain = X$ dB. The actual gains are calculated below:

$$FbAtten.Gain = 5 * \lceil (FbAtten1.Gain / 5) \rceil = 5 * \text{ceiling}(FbAtten1.Gain / 5)$$

$$DC.Gain = DC1.Gain + \text{mod}_5(FbAtten1.Gain)$$

Note: The ceiling function takes a number and returns the nearest integer that is greater than or equal to it.

So the range of values both components can attain are as follows:

$$FbAtten.Gain \in \{0, -5, -10, -15, -20, -25\}$$

$$DC.Gain \in \{Y, Y-1, Y-2, Y-3, Y-4\}$$

Note: The resulting gain control range in CMX998 is 0 dB to -29 dB.

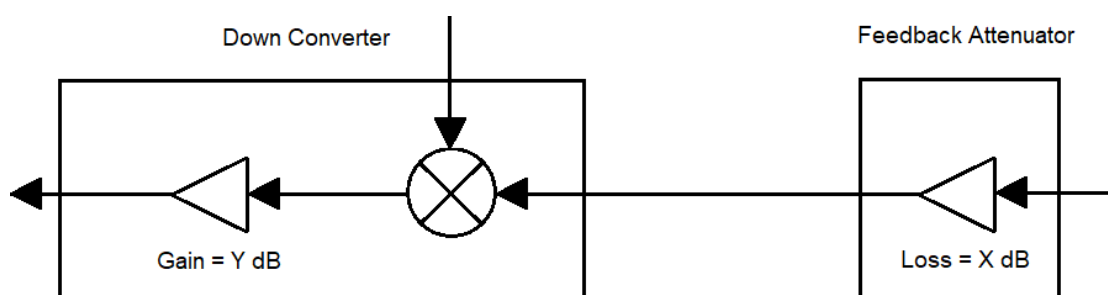


Figure 6: Diagram of the Feedback Attenuator and Down Converter of CMX998.

9 NOISE AND GAIN MORE GENERALLY

The noise factor for a cascade of N amplifiers is found using the Friis Formula:

$$F = F_1 + \text{SUM}(m=2:N)[(F_m - 1) / (\text{PRODUCT}(n=1:m-1)[G_n])]$$

For the forward path, we'll now define: $F_1 = \text{ErrAmp.Noise}$, $F_2 = \text{UC.Noise}$, $F_3 = \text{FwdAtten.Noise}$, $F_4 = \text{FwdDriver.Noise}$, $F_5 = \text{ExtAttenFwd.Noise}$, $F_6 = \text{ExtDriver.Noise}$, $F_7 = \text{PA.Noise}$.

So: $F_{fw} = F_1 + \text{SUM}(m=2:7)[(F_m - 1) / (\text{PRODUCT}(n=1:m-1)[G_n])]$

For the feedback path we define: $F_9 = \text{Coupler.Noise}$, $F_{10} = \text{ExtAttenFb.Noise}$, $F_{11} = \text{FbAtten.Noise}$, $F_{12} = \text{DC.Noise}$, $F_{13} = \text{FbAmp.Noise}$.

So: $F_{fb} = F_9 + \text{SUM}(m=10:13)[(F_m - 1) / (\text{PRODUCT}(n=9:m-1)[G_n])]$

The gain for a cascade of N amplifiers is found by multiplying all of the gains together:

$$G = \text{PRODUCT}(m=1:N)[G_m]$$

So for the forward path we'll define the gains as: $G_1 = \text{ErrAmp.Gain}$, $G_2 = \text{UC.Gain}$, $G_3 = \text{FwdAtten.Gain}$, $G_4 = \text{FwdDriver.Gain}$, $G_5 = \text{ExtAttenFwd.Gain}$, $G_6 = \text{ExtDriver.Gain}$, $G_7 = \text{PA.Gain}$, $G_8 = \text{CouplerFwd.Gain}$.

So: $G_{fw} = \text{PRODUCT}(m=1:8)[G_m]$

For the feedback path we define: $G_9 = \text{Coupler.Gain}$, $G_{10} = \text{ExtAttenFb.Gain}$, $G_{11} = \text{FbAtten.Gain}$, $G_{12} = \text{DC.Gain}$, $G_{13} = \text{FbAmp.Gain}$.

So: $G_{fb} = \text{PRODUCT}(m=9:13)[G_m]$

10 MODELLING THE NOISE

The noise power at the input of the forward path can be calculated as:

$$N_{fw} = (F_{fw} - 1) * k * T * B$$

Similarly the noise power referred to the input of the feedback path is:

$$N_{fb} = (F_{fb} - 1) * k * T * B$$

Calculations are provided in Appendix B on page 13.

*Note: $k * T * B$ is the thermal noise. This is considered to be the initial noise input into the system. $k = 1.380649 * 10^{-23}$ in the Boltzmann constant, $T = 290$ K is the temperature and B is the bandwidth (in Hz).*

We can model this system as we have in Figure 7 below:

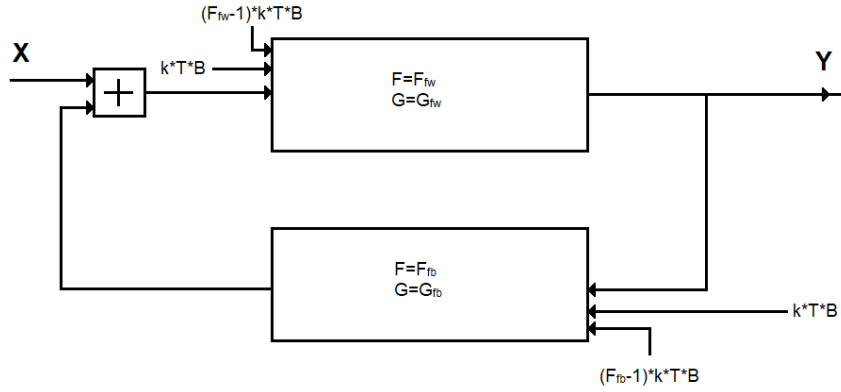


Figure 7: Block diagram of the CMX998 system.

This shows three inputs to the forward path block which consist of the signal from the summing junction, the initial noise term (kTB) due to the input match and the noise term due to the forward path. Similarly the feedback path has three inputs consisting of the signal from the output Y , the noise term due to the input match and the noise term due to the feedback path.

The total noise power applied to the input of the forward path is therefore:

$$N_1 = k*T*B + (F_{fw} - 1)*k*T*B = F_{fw}*k*T*B$$

And the total noise power applied to the input of the feedback path is:

$$N_2 = k*T*B + (F_{fb} - 1)*k*T*B = F_{fb}*k*T*B$$

Figure 8 below shows the noise sources applied to the inputs of these two blocks:

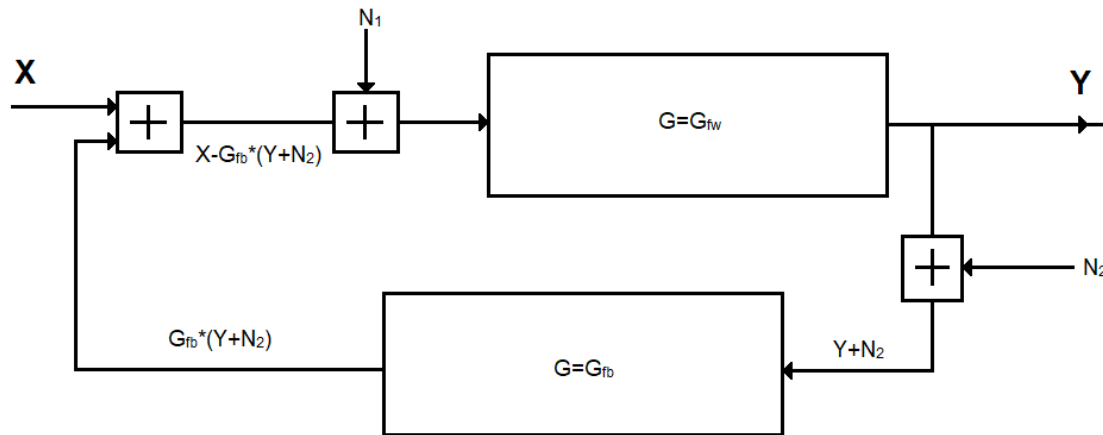


Figure 8: Block diagram for the noise in CMX998.

Note: Later on we will add a Phase Shift term, ϑ , which is introduced in the feedback path due to the LO and a time delay term, τ .

By analyzing the system we can obtain an expression for the output Y :

$$Y = (X - (Y + N_2)*G_{fb} + N_1)*G_{fw}$$

Rearranging this for Y we get:

$$Y = (X * G_{fw}) / (1 + G_{fw} * G_{fb}) + G_{fw} * (N_1 - N_2 * G_{fb}) / (1 + G_{fw} * G_{fb})$$

The first term gives the closed loop power gain of the system. The second term gives the noise power. We can therefore see that the gain of the system is:

$$\text{System.Gain} = G_{fw} / (1 + G_{fw} * G_{fb})$$

And the noise contribution of the loop is given by:

$$\text{Loop2.Noise} = G_{fw} * (N_1 + N_2 * G_{fb}) / (1 + G_{fw} * G_{fb})$$

Note: There has been a sign change. This is due to the noise terms being uncorrelated and independent so they are additive.

When adding in the Phase Shift, ϑ , and the time delay, τ , terms we obtain the following two expressions:

$$\text{System.Gain} = G_{fw} / (1 + G_{fw} * G_{fb} + 2 * \text{Re}(e^{i * (\vartheta - \tau * (f_c + f))}) * \text{sqrt}(G_{fw} * G_{fb}))$$

$$\text{Loop1.Noise} = \text{System.Gain} * (N_1 + N_2 * G_{fb})$$

Note: The term f_c is the centre frequency and f is the offset frequency.

This noise term predicts the noise of the loop inside the loop bandwidth. The forward path will determine the noise outside the loop bandwidth since the feedback path is essentially inoperative in this region. The error amplifiers will also be inoperative so the out-of-band (wideband) noise factor is given by:

$$F_{wb} = F_2 + \text{SUM}(m=3:7) [(F_m - 1) / (\text{PRODUCT}(n=2:m-1) [G_n])]]$$

And the associated gain is that which follows the Up-Converter:

$$G_{wb} = G_{fw} = \text{PRODUCT}(m=3:8) [G_m]$$

11 RESULTS: NOISE POWER SPECTRAL DENSITY

The Noise Power Spectral Density is usually expressed in dBm/Hz. Thus the wideband output noise is:

$$\text{Wideband.Noise} = -174 + 10 * \log_{10}(F_{wb} * G_{wb}) \quad \text{in dBm/Hz.}$$

*Note: $N_{floor} = 10 * \log_{10}(k * T * B / B) + 30 = -174 \text{ dBm/Hz}$ is the thermal noise which is the noise floor. The '+ 30' term converts from dB to dBm because $10 * \log_{10}(1000) = 30$.*

The noise contribution of the loop referred to the input is:

$$\text{Loop.Noise} = 10 * \log_{10}(\text{Loop1.Noise} / B) + 30 \quad \text{in dBm/Hz.}$$

$$= 10 * \log_{10}(\text{System.Gain} * k * T * (F_{fw} + F_{fb} * G_{fb})) + 30 \quad \text{in dBm/Hz.}$$

The open loop gain is:

$$\text{OpenLoop.Gain} = 10 * \log_{10}(G_{fw} * G_{fb}) \quad \text{in dB.}$$

The combined noise is:

$$\text{Overall.Noise} = 10 * \log_{10}(10^{(\text{Loop.Noise}/10)} + 10^{(\text{Wideband.Noise}/10)}) \quad \text{in dBm/Hz.}$$

Appendix A

Let's recall the image of the Up Converter:

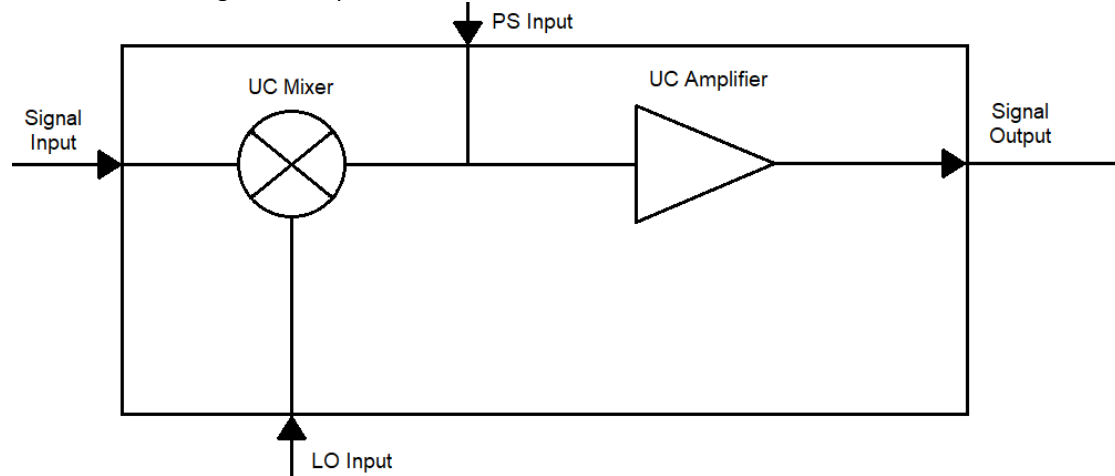


Figure 9: Diagram of the Up Converter in CMX998.

By definition the noise factor of a component, X, is:

$$F = (\text{Signal.Input/Noise.Input})/(\text{Signal.Output/Noise.Output}) \quad \text{or in shorthand:}$$

$$F = (S_{in}/N_{in})/(S_{out}/N_{out}) = (S_{in} * N_{out})/(S_{out} * N_{in})$$

To use this we need to think about what properties our component, X, has:

$$G_X = \text{ComponentX.Gain}$$

$$N_X = \text{ComponentX.Noise}$$

So the component has a given gain and adds some noise. Let's now calculate the noise factor:

$$S_{out} = G_X * S_{in} \quad \text{Signal output is the signal input multiplied by the gain.}$$

$$N_{out} = N_{in} * G_X + N_X \quad \text{Noise output is the Noise input multiplied by the gain plus the component noise.}$$

$$F_X = (S_{in} * N_{out})/(S_{out} * N_{in}) = (S_{in} * (N_{in} * G_X + N_X))/(G_X * S_{in} * N_{in}) = 1 + N_X/(G_X * N_{in})$$

From this we can derive an equation for the component noise:

$$N_X = (F_X - 1) * G_X * N_{in}$$

Now we can derive the noise factor of the whole up converter using the same method:

Note: We assume the mixer and Phase Shifter have no gain and the mixer is noiseless.

$$S_{out} = G_{mix} * G_{PS} * G_{UC} * S_{in} = G_{UC} * S_{in}$$

$$\begin{aligned} N_{out} &= N_{in} * G_{mix} * G_{PS} * G_{UC} + N_{LO} * G_{PS} * G_{UC} + N_{PS} * G_{UC} + N_{UC} \\ &= N_{in} * G_{UC} + N_{LO} * G_{UC} + N_{PS} * G_{UC} + N_{UC} \\ &= G_{UC} * (N_{in} + N_{LO} + N_{PS}) + N_{UC} \end{aligned}$$

$$\begin{aligned}
 F &= (S_{in} * N_{out}) / (S_{out} * N_{in}) = (S_{in} * (G_{UC} * (N_{in} + N_{LO} + N_{PS}) + N_{UC})) / (G_{UC} * S_{in} * N_{in}) \\
 &= (S_{in} * G_{UC} * (N_{in} + N_{LO})) / (G_{UC} * S_{in} * N_{in}) + (S_{in} * G_{UC} * N_{PS}) / (G_{UC} * S_{in} * N_{in}) + \\
 &\quad (S_{in} * N_{UC}) / (G_{UC} * S_{in} * N_{in}) \\
 &= (S_{in} * G_{UC} * (N_{in} + N_{LO})) / (G_{UC} * S_{in} * N_{in}) + (S_{in} * G_{UC} * (F_{PS}-1) * G_{PS} * N_{in}) / (G_{UC} * S_{in} * N_{in}) + \\
 &\quad (S_{in} * (F_{UC}-1) * G_{UC} * N_{in})) / (G_{UC} * S_{in} * N_{in}) \\
 &= (S_{in} * G_{UC} * N_{LO}) / (G_{UC} * S_{in} * N_{in}) + 1 + (F_{PS} - 1) + (F_{UC} - 1) \\
 &= N_{LO} / N_{in} + (F_{PS} - 1) + F_{UC} = (10^{((LO1.Noise + UC.Input.dB)/10)}) / N_{in} + (F_{PS} - 1) + F_{UC} \\
 &= 10^{(UC.Input.dB/10)} * (10^{(LO1.Noise/10)}) / N_{in} + (F_{PS} - 1) + F_{UC} \\
 &= UC.Input * (10^{(LO1.Noise/10)}) / N_{in} + (F_{PS} - 1) + F_{UC} \\
 &= UC.Input * F_{LO} + F_{UC} + (F_{PS} - 1) \\
 &= UC.Input * LO.Noise + UC1.Noise + (PhaseShifter.Noise - 1)
 \end{aligned}$$

where $LO.Noise = F_{LO} = (10^{(LO1.Noise/10)}) / N_{in}$

□

Appendix B

Let's recall part of our earlier image for the Cartesian Loop design:

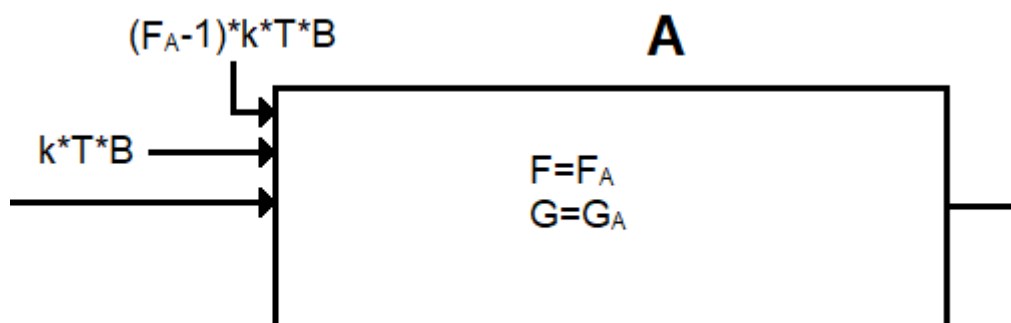


Figure 10: Diagram of an arbitrary block, A.

We will consider a generic block labelled A here but without loss of generality we could equally consider this as the forward or feedback blocks. We want to prove that the noise added by block A, N_A , is:

$$(F_A - 1) * G_A * k * T * B$$

and hence we can say the noise input to the forward path is: $(F_A - 1) * k * T * B$.

Note: Signal input, S_{in} , is arbitrary and the noise input is $N_{in} = k * T * B$.

So let's consider the noise factor of block A:

$$\begin{aligned}
 F_A &= (S_{in}/N_{in}) / (S_{out}/N_{out}) = (S_{in} * N_{out}) / (S_{out} * N_{in}) \\
 F_A &= (S_{in} * (G_A * N_{in} + N_A)) / (G_A * S_{in} * N_{in}) = (G_A * N_{in} + N_A) / (G_A * N_{in}) \\
 F_A &= 1 + N_A / (G_A * N_{in})
 \end{aligned}$$

$$F_A * G_A * N_{in} = G_A * N_{in} + N_A$$

Rearranging to find the noise added by block A:

$$N_A = (F_A - 1) * G_A * N_{in} = (F_A - 1) * G_A * k * T * B$$

So we can model the noise into block A as:

$$N_A / G_A = (F_A - 1) * k * T * B$$

□

Appendix C

Lemma: When we have a series of M cascaded attenuators the power gain of this series is equal to the product of each individual component's power gain.

Proof:

Let G_m be the power gain of the m^{th} attenuator where $m \in \{1, \dots, M\}$.

Recall that the gain of a cascade of M amplifiers (an attenuator is considered to be an amplifier with loss/negative gain) is:

$$G = \text{PRODUCT}(m=1:M)[G_m]$$

This proves our lemma.

□

Theorem: When we have a series of M cascaded attenuators the noise factor of this series is equal to the product of each individual component's noise factor.

Proof:

Let G_m be the power gain and F_m be the noise factor of the m^{th} attenuator where $m \in \{1, \dots, M\}$.

A property of Attenuators is that their noise factor is equal to the inverse of their gain, so: $F_m = 1/G_m$.

Let's assume $N=1$. $F_1 = F_1$ is trivial.

Now let $M \in \mathbf{N}$. Assume that for a cascade of M attenuators, $F = \text{PRODUCT}(m=1:M)[F_m]$.

Let's consider a cascade of M + 1 attenuators. By our assumption above we can model the first M attenuators as an attenuator with the gain $G_{1..M} = \text{PRODUCT}(m=1:M)[G_m]$ and noise factor $F_{1..M} = \text{PRODUCT}(m=1:M)[F_m]$. Let's also recall from Appendix B that the noise generated by a path X can be written as: $N_X = (F_X - 1) * G_X * N_{in}$. Equivalently: $N_X / (G_X * N_{in}) = (F_X - 1)$.

Let's now consider the noise factor of the whole series:

$$F = (S_{in}/N_{in}) / (S_{out}/N_{out}) = (S_{in} * N_{out}) / (S_{out} * N_{in})$$

$$\begin{aligned}
 &= (S_{in} * (N_{in} * G_{1..M} * G_{M+1} + N_{1..M} * G_{M+1} + N_{M+1})) / (G_{1..M} * G_{M+1} * S_{in} * N_{in}) \\
 &= 1 + N_{1..M} / (G_{1..M} * N_{in}) + N_{M+1} / (G_{1..M} * G_{M+1} * N_{in}) \\
 &= 1 + (F_{1..M} - 1) + (F_{M+1} - 1) / G_{1..M} \\
 &= F_{1..M} + (F_{M+1} - 1) / (1 / F_{1..M}) = F_{1..M} + (F_{M+1} - 1) * F_{1..M} = (1 + F_{M+1} - 1) * F_{1..M} \\
 &= F_{M+1} * F_{1..M} = F_{M+1} * \text{PRODUCT}(m=1:M)[F_m] \\
 &= \text{PRODUCT}(m=1:M+1)[F_m]
 \end{aligned}$$

□

Appendix D

Below is the structure of the written code.

Definitions

Input values for power gain and noise figure of the following components: Error Amplifiers, Up Converter, Attenuator 1, Driver 1, Attenuator 2, Driver 2, Power Amplifier, Coupler (Forward and Feedback paths), Down Attenuator, Down Converter, Feedback Amplifiers.

We also input the Phase Shifter noise figure and its shift angle (ϑ), the time delay (τ), the centre frequency, the signal output level and conditions for the LO noise profile, namely LO.N1, LO.N2 and the drop-off frequency.

We also want to know whether we are using an integrator or a low pass filter (assign value of 0 or 1) and which graphs are to be plotted (Create a vector of 1s and 0s for yes or no).

We also have some fixed inputs:

- Boltzmann constant: $k = 1.380649 \times 10^{-23}$ J/K
- Temperature: $T = 290$ K
- Capacitance values: $C_1 = C_2 = 100$ pF, $C_3 = 3.3$ nF (see CMX998_ds.pdf p10)
- Resistance values: $R_1 = R_2 = 1$ k Ω , $R_3 = 100$ k Ω , $R_4 = 150$ Ω , $R_5 = 15$ Ω (see CMX998_ds.pdf p10)

*Note: Instead of Capacitance and Resistance Input values we may input the time constants $t_1 = C_1 * R_1$, $t_2 = C_3 * R_4$ and $t_3 = C_3 * R_5$.*

Convert all dB values into non-dB values.

Calculations

Find the Up and Down Converter input powers by taking the output power and work back using the power gains.

Create a vector of frequencies that you'd like to plot. The plots are on a logarithmic scale so we create a vector which creates ninety evenly spaced points between each factor of 10.

Create a 'for' loop so that we can evaluate all the elements of our frequency vector.

LO: Perform the calculation for the LO noise graph (dBc/Hz), subsequently calculate the LO noise factor (LO.Noise).

CMX998 Loop Noise Calculator

H(s): Define $s=2\pi i*f$ and the three time constants (if they haven't already been defined in the definitions). Use an 'if' function to distinguish between the integrator and Low Pass Filter path. Calculate H(s).

Define our updated Error Amplifier gain (with H(s)) and the noise factor of the overall up and down converters.

Noise + Gain: Calculate the power gain and noise factor for the forward and feedback paths as well as the wideband values.

Calculate the system gain and noise generated by the loop.

Results

Create vectors which evaluate the Open-Loop gain, Wideband noise, Loop noise and Combined noise for each frequency point.

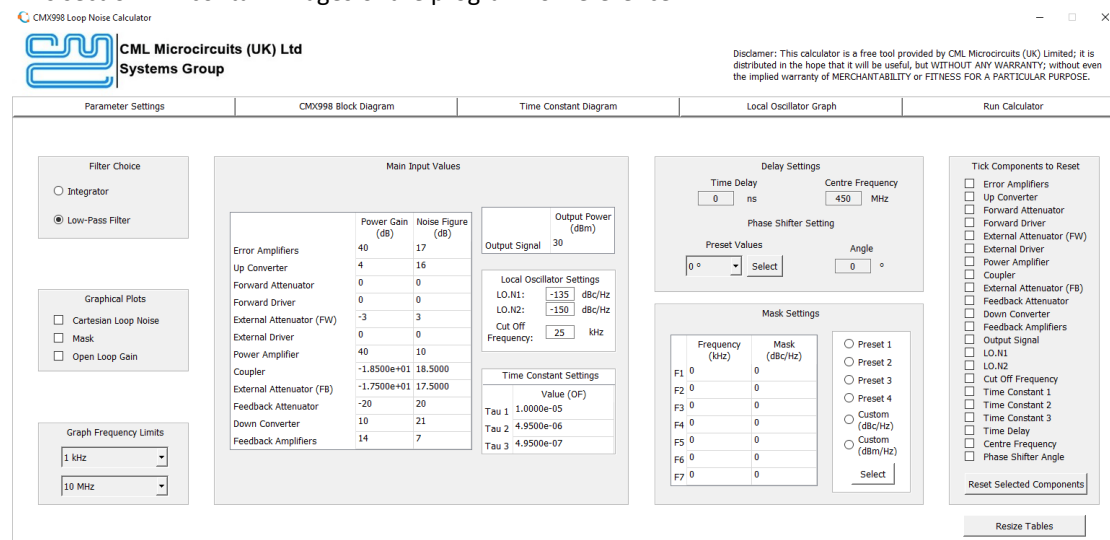
End the 'for' loop.

Plot the relevant graphs.

□

Appendix E

This section will contain images of the program for reference.



CMX998 Loop Noise Calculator

Parameter Settings | CMX998 Block Diagram | Time Constant Diagram | Local Oscillator Graph | Run Calculator

Filter Choice
☐ Integrator
☒ Low-Pass Filter

Graphical Plots
☐ Cartesian Loop Noise
☐ Mask
☐ Open Loop Gain

Graph Frequency Limits
 1 kHz
 10 MHz

Main Input Values

	Power Gain (dB)	Noise Figure (dB)	Output Power (dBm)
Error Amplifiers	40	17	30
Up Converter	4	16	
Forward Attenuator	0	0	
Forward Driver	0	0	
External Attenuator (FW)	-3	3	
External Driver	0	0	
Power Amplifier	40	10	
Coupler	-1.8500e+01	18.5000	
External Attenuator (FB)	-1.7500e+01	17.5000	
Feedback Attenuator	-20	20	
Down Converter	10	21	
Feedback Amplifiers	14	7	

Local Oscillator Settings
 LO.N1: -135 dBc/Hz
 LO.N2: -150 dBc/Hz
 Cut Off Frequency: 25 kHz

Time Constant Settings
 Value (OF)
 Tau 1 1.0000e-05
 Tau 2 4.9500e-06
 Tau 3 4.9500e-07

Delay Settings
 Time Delay: 0 ns
 Centre Frequency: 450 MHz
 Phase Shifter Setting: Preset Values, Angle: 0°

Mask Settings

Frequency (kHz)	Mask (dBc/Hz)
F1	0
F2	0
F3	0
F4	0
F5	0
F6	0
F7	0

Tick Components to Reset
☐ Error Amplifiers
☐ Up Converter
☐ Forward Attenuator
☐ Forward Driver
☐ External Attenuator (FW)
☐ External Driver
☐ Power Amplifier
☐ Coupler
☐ External Attenuator (FB)
☐ Feedback Attenuator
☐ Down Converter
☐ Feedback Amplifiers
☐ Output Signal
☐ LO.N1
☐ LO.N2
☐ Cut Off Frequency
☐ Time Constant 1
☐ Time Constant 2
☐ Time Constant 3
☐ Time Delay
☐ Centre Frequency
☐ Phase Shifter Angle

Reset Selected Components

Resize Tables

Figure 11: CMX998 Loop Noise Calculator User Interface – “Parameter Settings” tab.

CMX998 Loop Noise Calculator

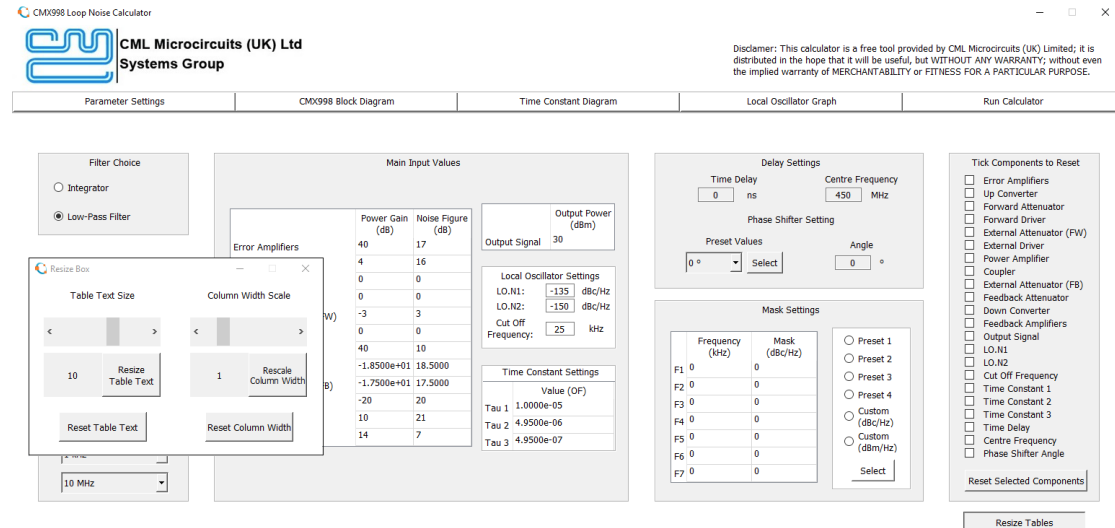


Figure 12: CMX998 Loop Noise Calculator User Interface – “Resize Tables button” pressed.

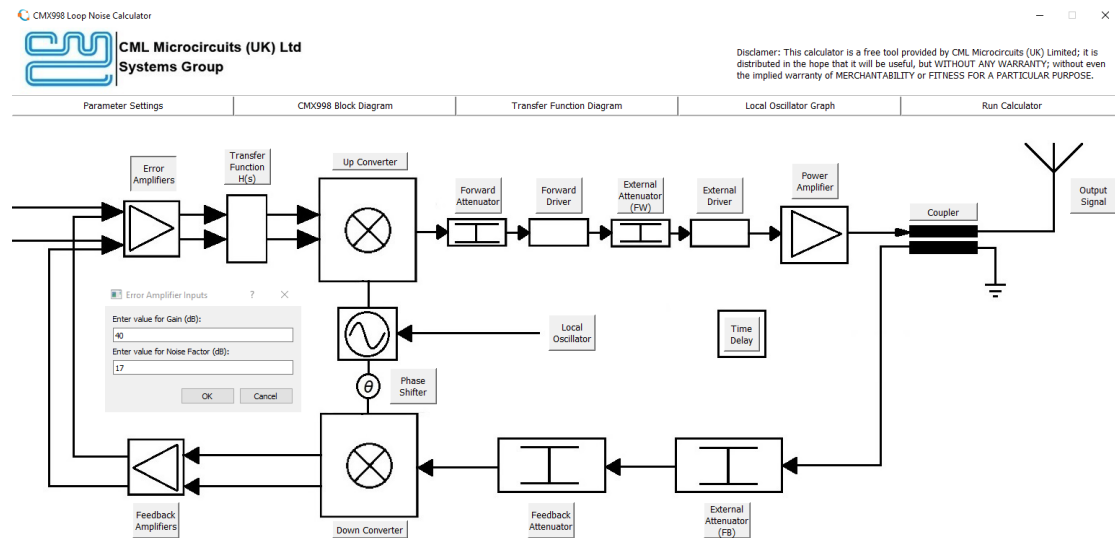


Figure 13: CMX998 Loop Noise Calculator User Interface – “CMX998 Block Diagram” tab.

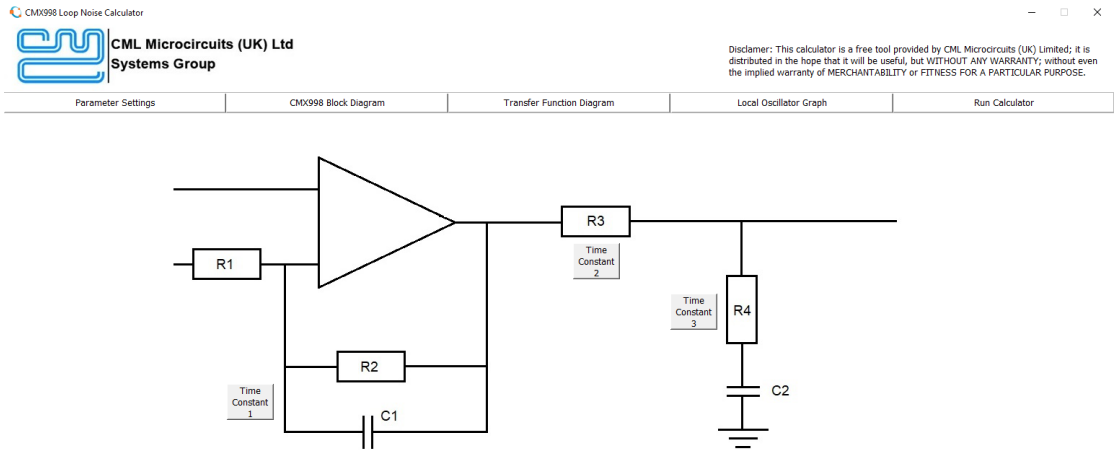


Figure 14: CMX998 Loop Noise Calculator User Interface – “Transfer Function Diagram” tab.

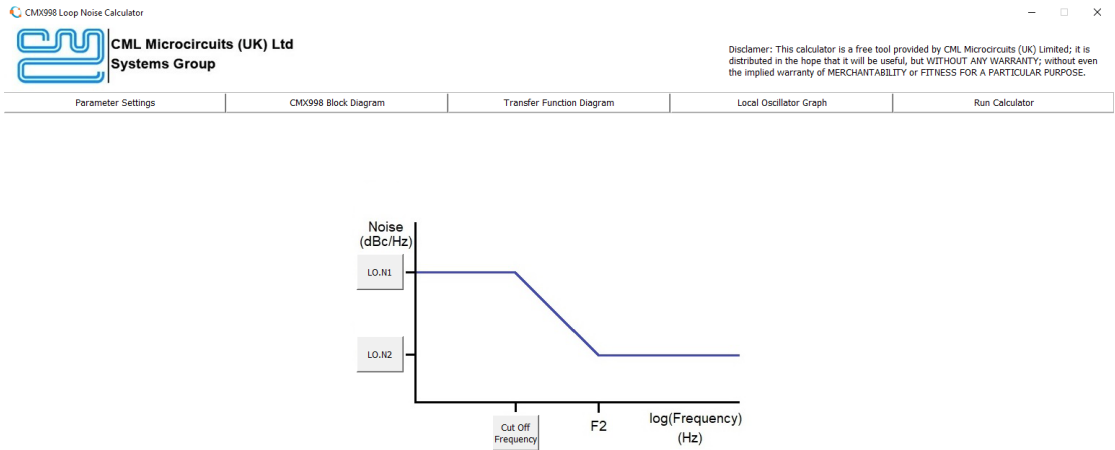


Figure 15: CMX998 Loop Noise Calculator User Interface – “Local Oscillator Graph” tab.

CMX998 Loop Noise Calculator

 CML Microcircuits (UK) Ltd
Systems Group

Disclaimer: This calculator is a free tool provided by CML Microcircuits (UK) Limited; it is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.

Parameter Settings CMX998 Block Diagram Transfer Function Diagram Local Oscillator Graph Run Calculator

Filter Choice

☐ Integrator

☒ Low-Pass Filter

Graphical Plots

☒ Cartesian Loop Noise

☒ Mask

☒ Open Loop Gain

Graph Frequency Limits

1 kHz

10 MHz

Main Input Values

	Power Gain (dB)	Noise Figure (dB)
Error Amplifiers	40	17
Up Converter	4	16
Forward Attenuator	0	0
Forward Driver	0	0
External Attenuator (FW)	-3	3
External Driver	0	0
Power Amplifier	40	10
Coupler	-1.8500e+01	18.5000
External Attenuator (FB)	-1.7500e+01	17.5000
Feedback Attenuator	-20	20
Down Converter	10	21
Feedback Amplifiers	14	7

Output Power (dBm): 30

Local Oscillator Settings

LO.N1: -135 dBc/Hz

LO.N2: -150 dBc/Hz

Cut Off: 25 kHz

Transfer Function Settings

Value (OP)

Time Constant 1: 1.0000e-05

Time Constant 2: 4.9500e-06

Time Constant 3: 4.9500e-07

Delay Settings

Time Delay: 0 ns

Centre Frequency: 450 MHz

Phase Shifter Setting

Preset Values: 0°

Angle: 0°

Mask Settings

Frequency (kHz)	Mask (dBc/Hz)
F1 100	-122
F2 250	-127
F3 500	-132
F4 5000	-142
F5 7000	-142
F6 8000	-142
F7 9000	-142

Preset 1

Preset 2

Preset 3

Preset 4

Custom (dBc/Hz)

Custom (dBm/Hz)

Tick Components to Reset

☐ Error Amplifiers

☐ Up Converter

☐ Forward Attenuator

☐ Forward Driver

☐ External Attenuator (FW)

☐ External Driver

☐ Power Amplifier

☐ Coupler

☐ External Attenuator (FB)

☐ Down Converter

☐ Feedback Amplifiers

☐ Output Signal

☐ LO.N1

☐ LO.N2

☐ Cut Off Frequency

☐ Time Constant 1

☐ Time Constant 2

☐ Time Constant 3

☐ Time Delay

☐ Centre Frequency

☐ Phase Shifter Angle

Reset Selected Components

Resize Tables

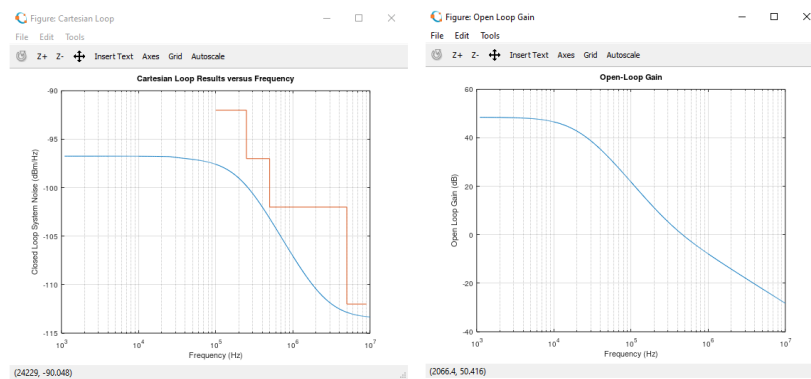


Figure 16: An example of the graphs displayed when the “Run Calculator” tab button is pressed for the values shown.